Indian Statistical Institute, Bangalore
B. Math (Hons.) First Year
Second Semester - Analysis II

Backpaper Exam — Date: 01st June 2026
Maximum marks: 100 — Duration: 3 hours

Each question carries 25 marks

1. (a) Prove that monotonic functions on $[a, b]$ are Riemann-integrable *(Marks 12)*.

   (b) If f is a nonnegative continuous function on $[0, 1]$ with $\int_0^1 f(x)dx = 0$, prove that $f = 0$ *(Marks 13)*.

2. (a) Let $f \in \mathcal{R}[0, 1]$ and $F(x) = \int_0^x f(t)dt$ for $x \in [0, 1]$. Prove that F is continuous. In addition if f is continuous, prove that F is differentiable and $F' = f$ *(Marks 13)*.

   (b) Determine all continuous functions f on $\mathbb{R}$ such that $2f(x) = \int_{-1}^1 f(x+t)dt$ for all $x \in \mathbb{R}$ and f attains its maximum on $\mathbb{R}$ *(Marks 12)*.

3. (a) Prove integration by parts for improper integral *(Marks 15)*.

   (b) If $f(x) = \sum a_n x^n$ has radius of convergence R, then prove that f is analytic in $(-R, R)$ *(Marks 10)*.

4. (a) If $f_n \to f$ uniformly on $[0, 1]$ and $f_n \in \mathcal{R}[0, 1]$, prove $f \in \mathcal{R}[0, 1]$ *(Marks 15)*.

   (b) Let (f_n) be a equicontiuous sequence of functions on $[0, 1]$ and (f_n) converges pointwise on $[0, 1]$. Prove that (f_n) converges uniformly on $[0, 1]$ *(Marks 10)*.